Experimental Study of Quantum Dynamics in a Regime of Classical Anomalous Diffusion

B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen
Department of Physics, The University of Texas at Austin, Austin, Texas 78712-1081
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We measure the momentum of cold cesium atoms in a periodically pulsed standing wave of light. This system is an experimental realization of the quantum kicked rotor. The momentum diffusion in the (chaotic) classical analog of this system is typically suppressed by quantum localization. We find, however, that for certain pulse amplitudes where the classical system exhibits anomalous diffusion, our momentum distributions are not exponentially localized. We observe a predicted correspondence between the classical and quantum energy growth as a function of pulse amplitude and period. [S0031-9007(98)07597-8]

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The correspondence between a quantum system and the underlying classical dynamics is a topic of fundamental importance. The paradigm system for the study of this correspondence is the kicked rotor, because of the simplicity of its equations of motion and the wealth of knowledge available on the classical system. One particularly interesting aspect of the kicked rotor is the existence of *accelerator modes,* which lead to Lévy flights in generic phase-space trajectories [1,2]. These Lévy flights can have a strong influence on the global transport properties of a system, and have been recently employed in the understanding of a subrecoil laser cooling scheme for atoms [3] and the motion of particles in a nonuniform fluid flow [4]. In the classical kicked rotor, the effects of the accelerator modes become most important in the long-time limit, because they account for a relatively small area in phase space. However, recent theoretical work has shown that these structures can have a dramatic effect in the quantum case, because of the nonlocal nature of the wave functions [5].

In this Letter, we study an experimental realization of the quantum kicked rotor, where cold cesium atoms are "kicked" by a periodically pulsed standing wave of far-detuned light. Previous work with sodium atoms has established this system as an excellent setting for the study of quantum chaos [6,7]. In this previous work, we observed *dynamical localization,* which is the quantum suppression of the classical chaotic momentum diffusion; the hallmark of this effect is a localized momentum distribution with an exponential profile. Here, we report the experimental study of the atomic momenta as a function of the pulse amplitude and period. We observe the oscillations in the momentum distribution widths that are expected from theory. We also observe that for certain kick amplitudes where accelerator modes are present in the classical phase space, the momentum distributions do not have the expected exponential form over the time scale of our experiment [8]. These results, which form the main focus of this Letter, suggest a correlation between classical anomalous diffusion and the observed quantum dynamics.

To describe our system, we begin with a two-level atom with transition frequency ω_0 interacting with a pulsed standing wave of near-resonant light of frequency ω_L. For sufficiently large detuning $\delta_L = \omega_0 - \omega_L$ (relative to the natural linewidth), the excited state amplitude can be adiabatically eliminated [9]. The atom can then be treated as a point particle. In this approximation, the center-of-mass motion of the atom is described by the Hamiltonian

$$H = \frac{p^2}{2M} + V_0 \cos(2k_L x) \sum_{n=1}^{N} F(t - nT). \qquad (1)$$

Here $V_0 = \hbar\Omega^2/8\delta_L$, k_L is the wave number, T is the pulse period, Ω is the resonant Rabi frequency, and $F(t)$ is a square pulse centered at $t = 0$ with duration t_p. We can rewrite (1) as a scaled, dimensionless Hamiltonian

$$H' = \frac{\rho^2}{2} - k\cos(\phi) \sum_{n=1}^{N} f(\tau - n), \qquad (2)$$

where $\phi = 2k_L x$, $\rho = (2k_L T/M)p$, $\tau = t/T$, $f(\tau)$ is a pulse of unit amplitude and scaled duration $\alpha = t_p/T$, $k = (8V_0/\hbar)\omega_r T^2$ is the scaled kick amplitude, $\omega_r = \hbar k_L^2/2M$ is the recoil frequency, and $H' = (4k_L^2 T^2/M)H$. In the quantized model, ϕ and ρ are conjugate variables satisfying the commutation relation $[\phi, \rho] = i\bar{k}$, where $\bar{k} = 8\omega_r T$ is a scaled Planck constant.

In the limit of arbitrarily short pulses, this system is equivalent to the δ-kicked rotor. The stochasticity parameter K completely specifies the classical δ-kicked rotor dynamics. For $K \gtrsim 1$, the classical dynamics are globally chaotic. For $K > 4$, the primary resonances become unstable, and the phase space is predominantly chaotic. In our system, the effective stochasticity parameter at zero momentum is given by the square-pulse expression $K = \alpha k$. The nonzero pulse widths lead to a reduction of K with increasing momentum.

In these experiments we study the evolution of the momentum distributions as a function of the two system parameters K and $\bar{k}$. Although the classical phase

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space is predominantly chaotic for $K > 4$, there are many small stable structures that influence the system dynamics, even for arbitrarily large K. In particular, when K is near $2\pi j$, where j is a positive integer, there exist stable accelerator modes that result from the periodicity of the phase space in momentum [10]. A trajectory inside one of these accelerator modes changes its momentum by $|\Delta\rho| = 2\pi j$ at each kick. (Note that other smaller accelerator modes exist for other values of K.) Additionally, trajectories that begin outside the accelerator modes will eventually wander near them and "stick" to their boundary for a possibly large but finite number of kicks [1]. This sticking results in the Lévy-flight trajectories mentioned above, where random-walk behavior is interspersed with strings of many correlated steps in the same direction. Thus, the accelerator modes change the global nature of phase-space transport from diffusion, characterized by random-walk trajectories, to anomalous diffusion, characterized by Lévy-flight trajectories. The diffusive behavior is described by a linear growth in energy, $E \equiv \langle (p/2\hbar k_{\mathrm{L}})^2/2 \rangle = Dt$, where D is the diffusion coefficient, whereas anomalous diffusion has a modified dependence $E = Dt^{\mu}$, where $\mu \neq 1$. A classical calculation of the diffusion coefficient shows an oscillatory dependence on K [10,11],

$$D(K) = \frac{K^2}{2}\left[\frac{1}{2} - J_2(K) + J_2^2(K) + O(K^{-3/2})\right], \tag{3}$$

where $J_2(K)$ is an ordinary Bessel function. This function, which is valid both in the absence of accelerator modes and for short times in the presence of accelerator modes, is plotted in Fig. 2. The maxima of this function coincide with the locations of the stable accelerator modes. Finally, we note that because of the finite nature of the pulses used in our experiment, the phase space is not exactly periodic in momentum, and hence our system does not have true accelerator modes. However, our system has "quasiaccelerator modes" [10], which behave like accelerator modes over a bounded region of phase space.

The quantum mechanical case, which applies to our experiment, exhibits behavior that is quite different from the classical case. In contrast to the long-time diffusion or superdiffusion of the classical case, the quantum system diffuses for only a short time and then stops when the momentum distribution reaches a characteristic exponential form [7]. Shepelyansky has predicted that the initial quantum diffusion rate D_0, and hence the characteristic length l of the localized distributions (the "localization length"), follows the classical case when K is replaced by K_{q}, with [12,13]

$$K_{\mathrm{q}} = K\left(\frac{2}{k\!\!\!^-}\right)\sin\left(\frac{k\!\!\!^-}{2}\right). \tag{4}$$

In this case $D_0(K) = D(K_{\mathrm{q}})/k\!\!\!^-{}^2$ and $l = 2\beta D_0$, where β has been determined to be $\sim 1/2$ via numerical simulations [12]. Notice that the zero crossings of (4) for integer $k\!\!\!^-/2\pi$ correspond to *quantum resonances*, where a plane wave at $p = 0$ undergoes ballistic growth in momentum, and exponential localization does not occur [7].

The quantum dynamics of the kicked rotor in the presence of accelerator modes has been studied theoretically by Hanson, Ott, and Antonsen [14]. In this work, the authors observed that any population contained within an accelerator mode would decay exponentially due to tunneling, and they developed a model for global momentum transport in the presence of accelerator modes. However, their simulations used small values of $k\!\!\!^-$, typically an order of magnitude smaller than those used in our experiment. A more recent theoretical investigation by Sundaram and Zaslavsky [5] focused on values of $k\!\!\!^-$ comparable to those used in our experiment as well as smaller values. In this work, the authors found evidence that accelerator modes enhance fluctuations in the localization length of the quasienergy states.

With these ideas in mind we have performed an experimental study of the quantum kicked rotor using cesium atoms. The experimental setup is similar to that of our earlier experiments on noise and dissipation in the quantum kicked rotor [15]. The experiments are performed on laser-cooled cesium atoms in a magneto-optic trap (MOT). Two actively locked single-mode diode lasers at 852 nm are used for cooling, trapping, and detection of the cesium atoms. Typically, we trap 10^6 atoms with $\sigma_x = 0.1$ mm and $\sigma_p/2\hbar k_{\mathrm{L}} = 4$. The trapping fields are then turned off, and the interaction potential is turned on. The pulsed standing wave is provided by a stabilized single-mode Ti:sapphire laser pumped by an argon-ion laser. This light passes through an acousto-optic modulator that controls the pulse sequence. The beam is aligned with the atoms and retroreflected through the chamber to form a standing wave. The beam has a typical power of 290 mW at the chamber and a waist of 1.44 mm. We detuned this beam 6.1 GHz to the red of the $(6S_{1/2}, F = 4) \rightarrow (6P_{3/2}, F = 5)$ cycling transition, with typical fluctuations of about 100 MHz. The pulse sequence consists of a series of 283 ns (full width at half maximum) pulses with a rise and/or fall time of 75 ns and less than 3 ns variation in the pulse duration. The pulse periods for the experiments presented here were varied from $T = 10$ to 60 μs, corresponding to the range $k\!\!\!^- \sim 1$ to 2π, with less than 4 ns variation per pulse period. The detection of momentum is accomplished by letting the atoms drift in the dark for a controlled duration (typically 15 ms). The trapping beams are then turned on in zero magnetic field, forming an optical molasses that freezes the position of the atoms [6]. The atomic position is recorded via fluorescence imaging in a short (10 ms) exposure on a cooled charge-coupled device (CCD). The final spatial distribution and the free-drift time enable the determination of the momentum distribution in the direction of the standing wave.

There are several systematic uncertainties in the experiment, which we now summarize. There are two systematic uncertainties in the determination of the momentum distributions. The first is the spatial calibration of the imaging system. The second arises from the ambiguity in the drift time due to motion occurring during different interaction times. These two factors give an overall systematic uncertainty of $\pm 4\%$ in the momentum measurements. The uncertainty in the stochasticity parameter K is $\pm 10\%$, with the largest contributions due to the measurement of laser beam profile and absolute power. The reduction in the effective value of K due to the nonzero temporal width of the pulses is only 6% out to $|p/2\hbar k_L| = 40$, and the effective value drops off by 25% at our maximum detectable momentum of $|p/2\hbar k_L| \approx 80$. Additionally, the measured distribution energies are affected by many factors because of their sensitivity to high momenta, and in the worst cases the absolute values may have systematic errors on the order of (20–30)%. These worst-case errors correspond to measurements where $K_q > 15$, since the distribution tails reach the edge of our detection system. In these cases, the quoted energy most likely underestimates the true energy. Finally, we note that our earlier experimental distributions had an exaggerated sharp bump at their center, because a fraction of the atoms, which we conservatively estimate to be around 20%, were in the $F = 3$ ground state [15]. These atoms did not interact as strongly with the standing wave, and so they did not diffuse nearly as far as the majority of the atoms in the $F = 4$ ground state. For the distributions shown in Fig. 1, we have ensured that nearly all the atoms were in the $F = 4$ ground state before the interaction with the standing wave by turning off the repump laser light 100 μs after the trapping laser light. For the energy data in Fig. 2, the $F = 3$ ground state is again around 20% populated; however, this mixed population leads only to a systematic reduction in the measured energy (on the order of 20%), and does not affect the locations of the observed peaks.

A typical exponentially localized momentum distribution for $K_q = 9.1$ is shown in Fig. 1(a). This distribution is what one would expect for dynamical localization. Figure 1(b), which corresponds to $K_q = 7.9$, shows a typical example of the observed distributions that are clearly not exponential. We observe the exponential distributions near the minima of $D(K_q)$, while nonexponential distributions like that of Fig. 1(b) occur near the maxima. Also, in the case of Fig. 1(b), it is interesting to note that the momentum distribution at short and intermediate times grows significantly more quickly than in the exponentially localizing case of Fig. 1(a). This behavior is especially evident from the "shoulders" on the distribution in Fig. 1(b) after 10 kicks, which are absent in the corresponding distribution in Fig. 1(a). At this time, we have no simple explanation for these distributions.

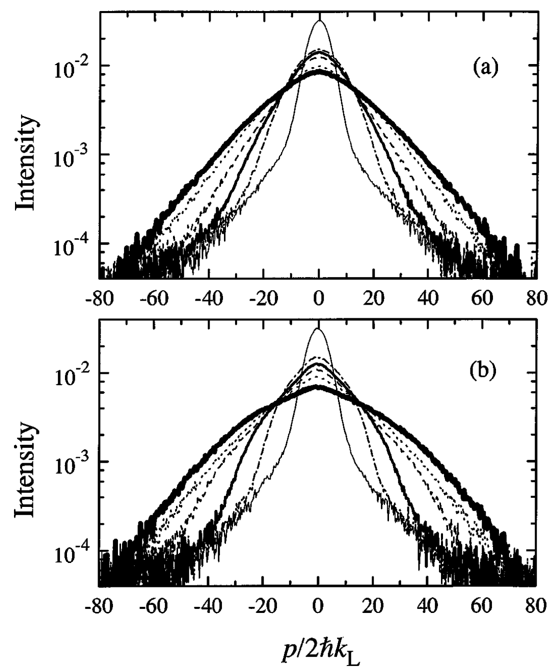

FIG. 1. Comparison of the momentum distribution evolution with $\bar{k} = 2.08$ for the cases (a) $K_q = 9.1$, (b) $K_q = 7.9$. Time steps shown are 0 kicks (light solid), 5 kicks (dash-dot), 10 kicks (bold), 20 kicks (dashed), 45 kicks (dotted), and 70 kicks (heavy bold). The vertical scale is logarithmic and in arbitrary units.

In order to observe the oscillations of the distribution widths versus K_q, we measured the distribution energies after 35 kicks as a function of K for several different values of $\bar{k}$. We note that although the theory is given in terms of localization length l, the presence of nonexponential distributions at the maxima of $D(K_q)$ make meaningful fits of the localization length questionable. However, the distribution energy E scales as l^2, so the energy is a sensible measure of the momentum distributions for comparison with theory. The results are shown in Fig. 2, where the energy is plotted against K_q. The classical diffusion curve from Eq. (3) is also shown for comparison. Because the maxima of the energy curves match for the different $\bar{k}$ cases, this data shows that these classical oscillations are present in the quantum kicked rotor and closely match the estimated quantum scaling factor in Eq. (4). The data in Fig. 2 that are "bunched" near $K_q = 0$ correspond to $\bar{k} = 6.24$, which is near the quantum resonance. In this case, as in our earlier work [7], we observe little growth in these momentum distributions. We also note that one may expect the energy data to have an overall growth as K_q^4 because, as noted above, the localization length l grows as K_q^2. However, we do not observe this scaling behavior in our experiment, and in fact our data has a similar overall growth to that of $D(K_q)$, which grows as K_q^2. This observed behavior is a result of the systematic effects described above, which have the strongest effect on the energies measured at larger K_q.

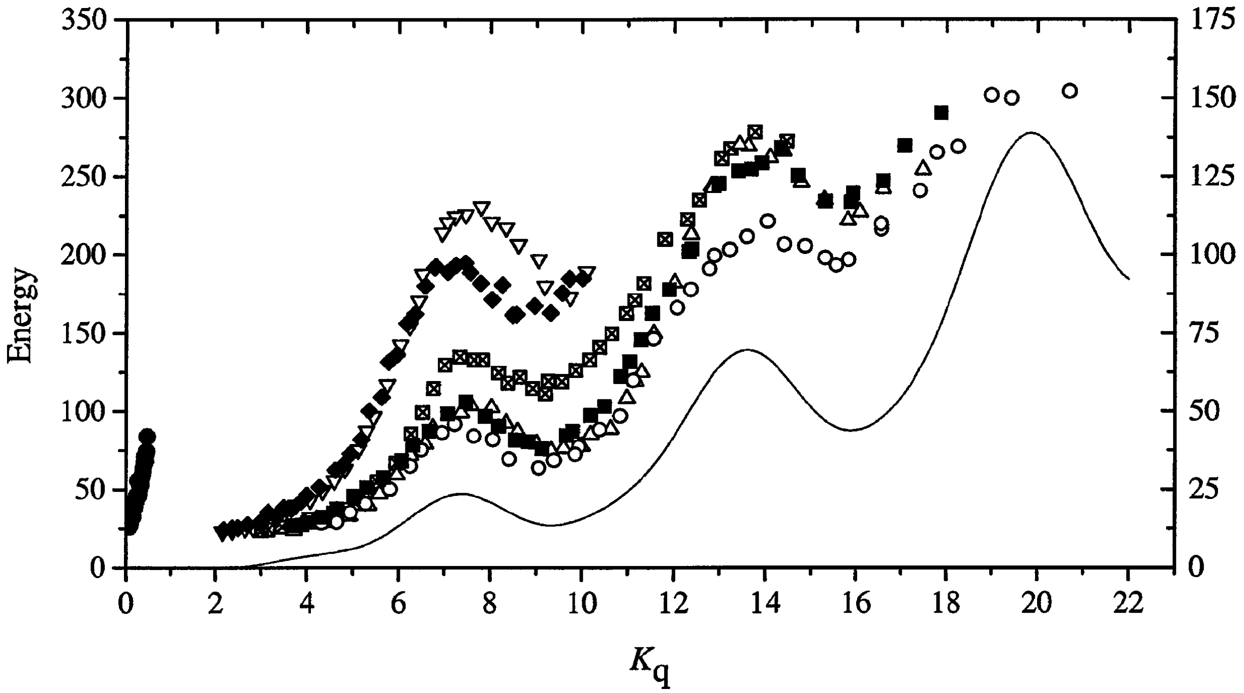

FIG. 2. Oscillations in the growth of energy as a function of K_q for various values of $\bar{k}$. Data show the average energy after 35 kicks for the cases $\bar{k} = 1.04$ (inverted triangles), 1.56 (crossed squares), 2.08 (triangles), 3.12 (open circles), 4.16 (filled squares); 28 kicks for $\bar{k} = 5.20$ (filled diamonds); and 24 kicks for $\bar{k} = 6.24$ (filled circles). The solid line is a plot of Eq. (3), the classical energy diffusion rate, showing the peaks related to anomalous diffusion. The correspondence of energy growth in the quantum case to the classical diffusion constant is consistent for all values of $\bar{k}$, supporting the validity of the scaling of the quantum kick strength K_q.

In conclusion, we have studied quantum transport in the quantum kicked rotor. We have observed the oscillatory dependence of the average energy growth on the kick strength and period. To our knowledge, this is the first experimental observation of these periodic variations in momentum transport in this system and the first experimental confirmation of the predicted quantum scaling of K. While the dependence of energy growth on K and $\bar{k}$ is in good qualitative agreement with theoretical expectations, the observed deviation of the momentum distributions for certain intervals in K from their expected, exponentially localized form was both surprising and interesting. We hope that this work will stimulate better theoretical understanding of the momentum distributions in the quantum kicked rotor.

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[1] G. M. Zaslavsky, M. Edelman, and B. A. Niyazov, Chaos **7**, 159 (1997).
[2] Joseph Klafter, Michael F. Schlesinger, and Gert Zumofen, Phys. Today, **49**, No. 2, 33 (1996).
[3] F. Bardou, J. P. Bouchaud, O. Emile, A. Aspect, and C. Cohen-Tannoudji, Phys. Rev. Lett. **72**, 203 (1994).
[4] T. H. Solomon, E. R. Weeks, and H. L. Swinney, Phys. Rev. Lett. **71**, 3975 (1993).
[5] Bala Sundaram and G. M. Zaslavsky (to be published).
[6] F. L. Moore, J. C. Robinson, C. Bharucha, P. E. Williams, and M. G. Raizen, Phys. Rev. Lett. **73**, 2974 (1994); J. C. Robinson, C. Bharucha, F. L. Moore, R. Jahnke, G. A. Georgakis, Q. Niu, M. G. Raizen, and Bala Sundaram, Phys. Rev. Lett. **74**, 3963 (1995).
[7] F. L. Moore, J. C. Robinson, C. F. Bharucha, Bala Sundaram, and M. G. Raizen, Phys. Rev. Lett. **75**, 4598 (1995).
[8] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, in Proceedings of the 1998 International Quantum Electronics Conference, 1998 (Report No. QPD9).
[9] R. Graham, M. Schlautmann, and P. Zoller, Phys. Rev. A **45**, R19 (1992).
[10] A. J. Lichtenberg and M. A. Lieberman, *Regular and Chaotic Dynamics* (Springer-Verlag, New York, 1992).
[11] A. B. Rechester and R. B. White, Phys. Rev. Lett. **44**, 1586 (1980); A. B. Rechester, M. N. Rosenbluth, and R. B. White, Phys. Rev. A **23**, 2664 (1981).
[12] D. L. Shepelyansky, Physica (Amsterdam) **28D**, 103 (1987).
[13] Doron Cohen, Phys. Rev. A **44**, 2292 (1991).
[14] James D. Hanson, Edward Ott, and Thomas M. Antonsen, Jr., Phys. Rev. A **29**, 819 (1984).
[15] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. **81**, 1203 (1998).